# Topic Study Group 39

## Language and Communication in Mathematics Classroom

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### 1. Core Topics and Research Interests for TSG-39

To begin with, we give a brief overview of the thematic foundations and challenges of TSG-39.

The variety of research shared by the presenters at the ICME conference makes the wide range of individual approaches to theory and methodology visible that research in language and communication in mathematics education is based on. Morgan (2006) refers to this focus to the role and nature of language and communication in relation to the learning of mathematics as a 'turn to language', although Pimm (2018) describes a long history of research connecting language within mathematics education. But a recognizable shift in the recognition of the complexity of the relationship between language and the learning of mathematics is taking place.

The research results presented in TSG-39 are based on an understanding of language and communication in a broad sense. The authors presented work focusing on the following fields of research: classroom interactions; interactions between children at play; multimodal analysis; as well as research that focuses on the multi-semiotic nature of mathematical activities, or even the role of silences (e.g., Boistrup and Samuelsson, 2018; Elliott and Ingram, 2016; O'Connor, Michaels, Chapin, and Harbaugh, 2017). Judith Moschkovich (2018) formulated four recommendations concerning the research of language and communication in mathematics education at ICME-13:

(1) using interdisciplinary approaches;
(2) building on existing methodologies;
(3) defining central constructs;
(4) recognizing central distinctions while avoiding dichotomies.

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With respect to points (1) and (2) of Moschkovich, it can be stated that research in our field is influenced from a variety of theoretical and methodological ideas from mathematics education as well as from other fields such as sociology, psychology, anthropology, linguistics, semiotics and many more. Working at the intersection of theories of learning and teaching mathematics and theories of language, interaction and communication is fundamental to doing research in mathematics education with a focus on language, interaction and communication. The connection of existing theories and methodologies is equally important as the development from new theories to drive innovation. A particular challenge when combining theories from different disciplines is on (3) defining terms and central constructs such as language, register and discourse and also to use them in a way that is consistent with the approach to research being taken. Finally, we turn to the recommendation of (4) recognising central distinctions while avoiding dichotomies. Moschkovich emphasises the often-made dichotomy between quantitative and qualitative approaches in contrast to the many examples of work that combines both. Another aspect of the dichotomy that should be considered in more detail is the type of attention given to language in research. However, language and mathematical activities are often intertwined, making it difficult to distinguish whether language is the focus of research or the medium through which researchers can access mathematical thinking or learning (Andersson & Wagner, 2019). The recommendations of Moschkovich (2018) shed light on the topic of mathematics and language from very different perspectives and were addressed in the various papers and posters as well as invited lectures.

## 2. Presentations in TSG-39

ICME-14 brought together researchers from around the world and TSG-39 included a range of researchers from 13 different countries, drawing on a range of different theoretical perspectives, different methodological approaches and with different focuses. Due to the Covid-19 pandemic, the meeting was held in a hybrid format. Part of the presentations were given online via Zoom, and another part at the location of ICME-14, East China Normal University, Shanghai, China in a lecture hall. A total of 21 papers and 7 posters were accepted for presentation in TSG-39. The papers were presented in 3 sessions over 3 days. In addition, two sessions were dedicated for one invited talk each (see Tab. 1 on the next page).

### *2.1. Invited talks*

The first invited talk[1] is given by Krummheuer from Kassel, Germany and Schütte from Leibniz University Hannover, Germany, and the second invited talk[2] is given by Herbal-Eisenmann from Michigan State University, USA and Ingram from Oxford University, UK. These invited talks enabled TSG participants to reflect on new broader perspectives regarding mathematics and language and to discuss the insights gained on this topic.

Tab. 1. The list of papers presented

| **Paper and author(s)** |
|---|
| ***Invited talks*** |
| [1] Cooperation, argumentation and learning — Three basic concepts referring to everyday procedures in teaching-learning situations in mathematics classes. ***Götz Krummheuer*** *and* ***Marcus Schütte*** (Germany). |
| [2] A political look at math communication. ***Beth Herbal-Eisenmann*** (USA) *and* ***Jenni Ingram*** (UK). |
| ***Session 1*** |
| [3] Meeting the challenges of research language and communication in mathematics education. ***Jenni Ingram*** (UK)*, Marcus Schütte* (Germany)*, Fengjuan Hu* (China)*, Máire Ni Riodáin* (Ireland)*, and Tran Vui* (Vietnam). |
| [4] Lifeworld connections in mathematics education — unquestioned, indispensable, and undefined? ***Elisa Bitterlich*** (Germany). |
| [5] The threshold of multiple representations for students to discover possible solutions for communicating their new ideas in integrated closed-open approach. ***Tran Vui*** (Vietnam). |
| [6] The practice and examination of opportunities to translate representation through problem-solving. ***Kunihiko Shimizu*** (Japan). |
| [7] Tau Ke: a software solution for capturing multiple representations of pangarau (mathematics) language. ***Piata Allen*** (Australia). |
| [8] The effects of using a modified Frayer Model to teach mathematics vocabulary to junior-form English learners in a Chinese medium-of-instruction secondary school. ***Wing-kwan Li*** *and Simon S. Y. Cha* (Hong Kong SAR, China). |
| [9] It always equalled an odd number: observing mathematical fluency through students' oral responses. ***Katherin Cartwright*** (Australia). |
| [10] Achieving meaningful statistics classroom learning through bilingualism and multilingualism: a case of selected grade 10 students in Marikina city. ***Mary Jane A. Castilla*** *and Catherine P. Vistro-Yu* (Philippine). |
| ***Session 2*** |
| [11] Discourse as the place for the development of mathematical thinking through an interactionist perspective. ***Judith Jung****, Marcus Schütte, and Götz Krummheuer* (Germany). |
| [12] Language-responsive support of meaning-making processes for understanding multipli-cative decomposition strategies. ***Annica Baiker*** *and Daniela Götze* (Germany). |
| [13] A study on the evaluating of learning opportunities in mathematics classes of secondary schools based on discourse analysis techniques. ***Zhihui Chen*** *and Yuting Tong* (China). |
| [14] Mathematical expression in different languages: The need for systematic description. ***Cris Edmonds-Wathen*** (Australia). |
| [15] A comparative study on teaching language of algebra classroom between novice teachers and expert teachers taking linear equation in one unknown as an example. ***Si-kai Wang*** *and Li-jun Ye* (China). |
| [16] Interactional obligations for collective argumentation in pair and group work. ***Rachel-Ann Böckmann*** *and Marcus Schütte* (Germany). |
| [17] How pre-service primary teachers engage in language responsive mathematics teaching while working on a scriptwriting task. ***Victoria Shure*** *and Bettina Rösken-Winter* (Germany). |
| [18] Support Systems as intersubjective processes between Teachers and Students. ***Ann-Kristin Tewes*** (Germany). |
| ***Session 3*** |
| [19] Epistemic (In)justice in mathematical communication between teachers and students. ***Lauren Hickman McMahon*** (USA). |
| [20] Identifying language demands for understanding the meaning of similarity. ***Kirstin Erath*** (Germany). |
| [21] Exploring a teacher's enactment of explanatory communication in a mathematics lesson. ***Fatou Sey*** (South Africa). |
| [22] Dissent and consensus situation structures in mathematics and computer science learning environments. ***Peter Ludes-Adamy*** *and Marcus Schütte* (Germany). |
| [23] Quadrilateral woop-de-doos: Language use and geometric property development of two fifth graders in a dynamic geomtetry learning environment. ***Candace Joswick*** *and Michael T. Battista* (USA). |

### *2.2. Session 1*

In Session 1 Bitterlich[4] focusses on situations with a lifeworld connection within mathematics lessons. Via interactional analyses and the analysis of linguistical markers her study aims at reconstructing the consequences of lifeworld connections on language use and the negotiation of (mathematical) meaning. Bitterlich underlines that lifeworld connections are frequently posed by the teacher, but seemingly seldom reflected concerning the underlying mathematical content. Shimizu[6] reports research that children will be proactive in their use of diverse mathematical representations when they have questions and explorative tasks in learning problem-solving. Based on this classroom practice, the process of representation was translated by exploring student questions, while students' feelings concerning their approach to mathematics were also important. Further, as students' inquiries deepened, their representation gradually became more sophisticated, and in the process, along with trial and error, a process of returning to representing thought was seen. Allen'***s*** study[7] focused on the use of digital technology in Māori-medium schools as a way of supporting Māori language, Māori knowledge and the acquisition of school mathematics. Following a period of Indigenous language and culture loss, in Aotearoa New Zealand, there has been rapid development of a corpus of mathematics terms and language to enable the teaching of mathematics in the Indigenous language, Māori. Allen's paper highlights the questions and concerns that continue to be raised, about the role of Indigenous mathematical practices in modern schooling. Li and Cha[8] conducted a study about learning Mathematics vocabulary using a Modified Frayer Model in a local secondary school. The model, also a graphic organiser, included four components which were specific to Mathematics: Mathematics symbols, diagrams or pictures, related vocabulary and sample sentences. The results showed that the model not only expanded the participants' Mathematics vocabulary, but also helped them remember it.

Cartwright[9] reports from a deductive analysis of student narrative data (transcripts) and student group work samples (artefacts) aiming to discover what characteristics students displayed both orally and written as evidence of mathematical fluency. She presents the oral and written language features students employed to explain their method and justify their strategies when solving mathematical tasks. Regarding mathematical fluency, Cartwright discusses that students' oral explanations were either procedurally-driven or findings driven. Students that were findings-driven could be identified at a higher level of fluency based on their ability to shift from low to high modality language. Cartwright proposes the need to analyze the language features (as a representation mode) of students' responses–particularly oral responses–as they provide data that might usually be missed or not present within written numerical work samples. Finally, Castilla and Vistro-Yu[10] examined the linguistic interactions that took place in a Statistics classroom using the framework of the second generation Cultural Historical Activity Theory (CHAT) with bilingualism and multilingualism as a potential primary CHAT construct. They looked into how the varying roles of students' alternating use of the Filipino and English languages

combined with their mathematical language contributed to the students' participation in the activity.

### *2.3. Session 2*

In Session 2 Jung et al.[11] focus on early childhood mathematics learning. Based on interactionist perceptions of mathematical learning, the development of mathematical thinking is described as increasing participation in mathematical discourses. For a more detailed description of these discourses, the so far common focus of interactionist approaches to mathematics learning on the analysis of mathematical negotiation of meaning is expanded to include a description of emerging argumentative structuring of the mathematical negotiation processes. Baiker and Götze[12] present a study investigating the impact of a language-responsive intervention on students´ understanding of decomposition strategies. Three second grade primary school teachers introduced multiplication ($n = 66$) by using meaning-related phrases of unitizing (e.g., '6 times 4 means 6 fours'), whereas three other classes taught without this focus served as control group ($n = 58$). The analyses of a multiplication post-test and a follow-up test showed a deeper understanding of decomposition strategies of the intervention group children.

Chen and Tong[13] addresses the question of the development of literacy skills of senior high school students in China and how to promote students' skills based on the model of mathematical core competencies in the classroom. Discourse analysis techniques are used to analyze two video-based lessons with different teaching methods. The results show that the evaluation model based on the idea of learning opportunity is reliable in terms of how students can benefit from the mathematical tasks and interaction (initiated questioning and feedback) provided by the teacher in the teaching process. Edmonds-Wathen[14] described a need for more systematic description of the variation in mathematical expression in different languages and the observed or speculated effects of this variation on mathematics education in those different languages, proposing a functional typology approach where languages are classified according to structural similarities and differences. Wang and Ye[15] reported a comparative study about the teaching language of a novice teacher and an expert teacher in algebra instruction. They classified the teacher's teaching language from the perspective of pragmatics, on the basis of which they specifically discussed the similarities and differences between the two teachers' use of teaching language.

Böckmann and Shütte[16] describes interactional obligations for bringing forth warrants or backings within collective argumentations which occur in social interactions of students working collaboratively in multi-age groups. She presents three interactional obligations — contradictions, mistakes and certain types of questions — as well as discusses how students' interpretation of interactional obligations can change within an interaction. Shure and Rösken-Winter[17] report on the results of a scriptwriting task study aimed at examining how pre-service primary mathematics teachers enrolled in a Master's program address language difficulties to support

students in gaining mathematical reasoning competencies. They present differences in practices between higher and lower performing pre-service teachers and discuss the study's relevance for teacher education. At the end Tewes[18] focusses on different support systems between students, primary school teachers and special needs education teachers. She therefore located support systems between the participants of the interaction and describes them as intersubjective processes which are designed together. The aim of the study is to reconstruct the potential effects of these support systems for the participation in inclusive mathematic lessons.

### *2.4. Session 3*

The Session 3 begins with McMahon[19] who describes in her paper, two forms of epistemic injustice offered by Fricker (2007) — testimonial and hermeneutic. Using a real-world example, she considers how such injustice can manifest itself in teacher-student communication about mathematics and discusses features of mathematical knowledge and skill that are necessary for children to experience epistemic justice in their interactions with teachers. Erath[20] reports from a Design Research study aiming at developing a language-responsive teaching-learning arrangement for the geometrical topic of similarity with a particular focus on supporting students' interaction in phases of unmoderated group work. She presents and discusses identified discourse practices and language means alongside a intended sequence of larger steps in the process of knowledge construction. Ludes-Adamy and Schütte[22] report on their research on learning environments with core topics of mathematics and computer science are examined. In the focus of digitalization, this topic and its connection to mathematics will play an important role in future curricula, making it an interesting object of investigation. Ludes-Adamy and Schütte present an ongoing study that examines, how the topic of computer science connected to mathematics learning can be approached in primary schools and what and how meanings are negotiated. They focus on the question what roles consensus and dissent play in interactional processes of negotiation and how the learning of the fundamentally new occurs in collective argumentation between pupils. Joswick and Battista[23] use a longitudinal analysis to track the language and geometry concept development of two 5th grade students regarding dynamic geometry for the study of quadrilaterals. In their paper, the authors describe the students' initial use of the term "whoop-de-doo" in their reasoning about quadrilateral shapes and point to the importance of our findings for productive classroom discourse.

## 3. Summary and Prospect

The papers that have been presented during our TSG present a wide range of research perspectives. They are published in the HAL open archive and can be freely accessed. The challenges that researchers in language, interaction and communication in mathematics education encounter, can also be seen as opportunities to foster innovation and influence teaching and learning of mathematics in a variety of contexts. Working

together as a Topic Study Group at ICME, discussing and connecting the differences and similarities in the research, enabled us all to take advantage of these opportunities and develop the field further. This collaboration will continue due to an ever-growing core of collaborating scientists in the field of mathematics, language and communication at the upcoming international meetings, such as those of CERME-2022, ETC-2022 and the following ICME meeting

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